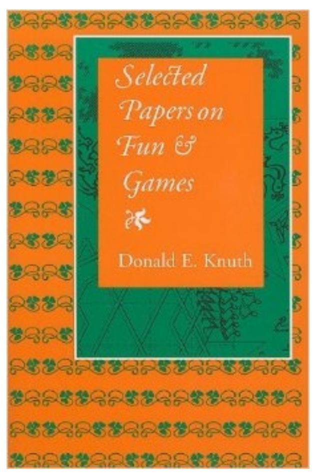
Donald E. Knuth

Donald E. Knuth*

Donald Ervin Knuth is a computer scientist, mathematician, and professor emeritus at Stanford University (www-cs-faculty.stanford.edu/~knuth). His work has revolutionized the entire computer world, and his famous multi-volume work *The Art of Computer Programming* is the programmers' Bible. Knuth created the modern infrastructure of typefaces along with the TeX system for computer typesetting and the METAFONT system for creating and rendering fonts. Algorithms and their analysis are his absolute domain, but Knuth is equally strong in many other areas, like literature and music, and puzzles are one of them.

Knuth's deep interest in puzzles of different kinds is interwoven naturally and with elegance into his computer scientific and mathematical work. The volumes of *The Art of Computer Programming* describe and discuss hundreds of puzzles (many of which are quite hard) in connection with writing programs to solve them efficiently by computer. In 2001 Knuth prepared several very detailed indexes to works of the world's most famous puzzle masters, Sam Loyd, Henry E. Dudeney, and Professor Louis Hoffmann (Angelo John Lewis). It is a precious resource for serious puzzlers, where they can find the origin of many famous puzzles.

One of Knuth's books, *Selected Papers on Fun and Games* (2010), is fully devoted to puzzles and similar topics. It is a must-have-and-read for all puzzle lovers. In its 49 chapters, this 760-page volume comprises hundreds of Knuth's original puzzle creations in different themes: articles in MAD Magazine, numbers, music, geometry, road signs, word play, knight's tours, car plates, games, and art, to name a few. The book is richly illustrated and shows amusing results not published elsewhere.



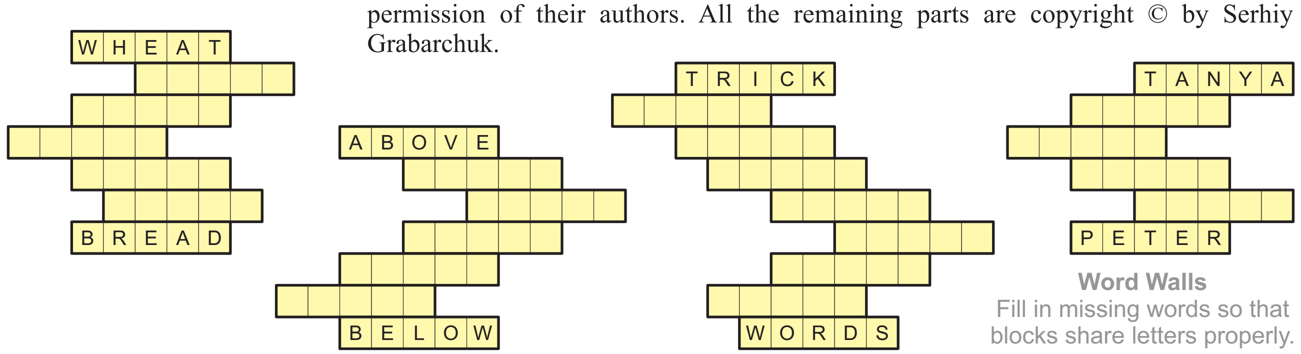
Selected Papers on Fun and Games by Donald Knuth. This book presents hundreds of Knuth's original puzzle creations in different themes.

Word Walls

This puzzle is a take off on Lewis Carroll's Word Ladders that Donald Knuth played with in 1994. In Knuth's puzzle, bricks of the ladder grid are shifted left and right so that each brick shares not one but *several* letters with the one underneath (or above it) exactly where they are adjacent. What an ingenious evolution of the classic challenge! Can you fill in missing words so that vertically neighboring bricks share letters properly?

-----

*) I am grateful to Donald Knuth, Stan Isaacs, George Miller, Kate Jones, and Pavel Curtis for their help with some puzzle materials for this article. Copyright © to puzzles stays with their respective authors. Copyrighted materials are used here with the kind permission of their authors. All the remaining parts are copyright © by Serhiy Grabarchuk.

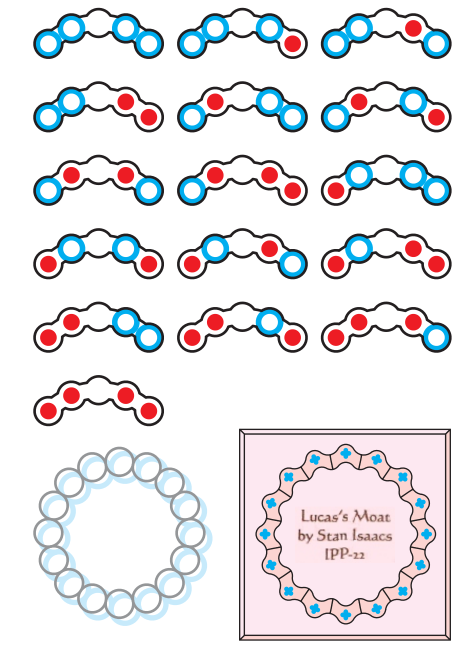


Word Walls
Fill in missing words so that blocks share letters properly.

Lucas's Moat

by Donald Knuth and George Miller

This puzzle is based on Knuth's adaptation of Édouard Lucas's original idea about cyclic sequences created from 0's and 1's. Here is a simplified representation of that idea. Take sixteen thin transparent *one-sided* (!) pieces shown on the right and string them together in a circle of length 16, overlapping in such a way that blue rings never coincide with red dots, and vice versa. Rings and dots can both cover empty spots in the pieces, though. Knuth's idea was transformed by George Miller into an elegant manipulative puzzle with waves and fish. Then Miller made it for Stan Isaacs who used it as his exchange gift at IPP22 in 2002 in Antwerp, Belgium.

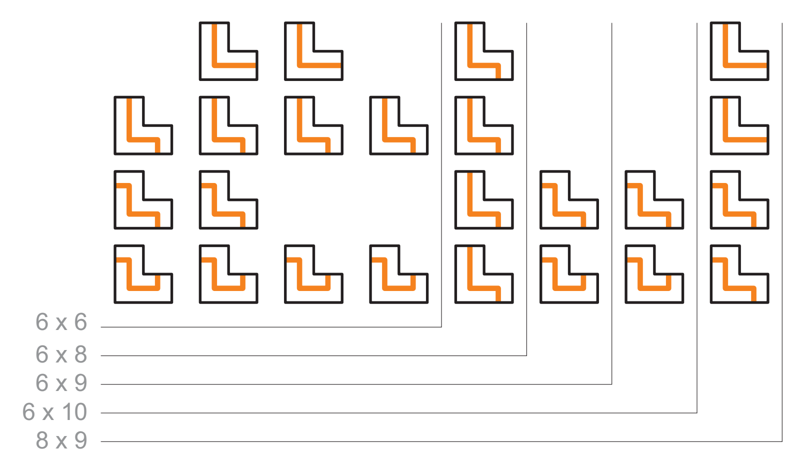


Lucas's Moat
String all sixteen transparent pieces in a circle so that rings and dots never coincide.
The sample on the right was developed and made by George Miller, www.puzzlepalace.com.
Stan Isaacs used it as his exchange gift at IPP22 in 2002 in Antwerp, Belgium.

Tromino Trails

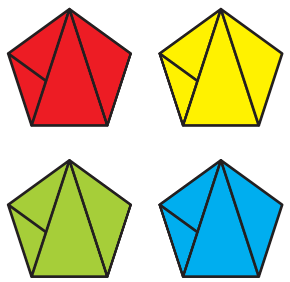
by Donald Knuth and Pavel Curtis

Donald Knuth created this puzzle in 2009 as a set of twenty-four two-sided triominoes (trominoes) each one with a fragment of a path depicted on both of its sides. The object is to form five different rectangles (6 x 6, 6 x 8, 6 x 9, 6 x 10, and 8 x 9), using increasing subsets of pieces. On each rectangle fragments of the path should make a single closed loop. Each rectangle has a unique solution. Pavel Curtis developed a clever changeable tray and a special marking on the pieces and tray for easy arrangements for each challenge. This puzzle was Stan Isaacs's exchange gift at IPP31 in 2011 in Berlin, Germany.

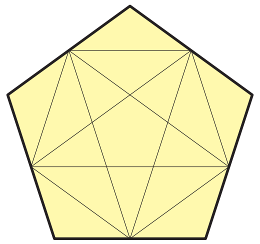


Tromino Trails
Form five rectangles with a single closed loop each.

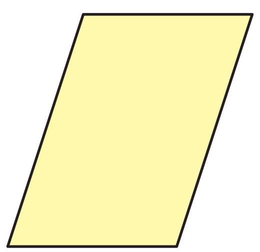
Tromino Trails
This perfect presentation of Knuth's puzzle was developed and precisely manufactured by Pavel Curtis, www.pavelspuzzles.com.



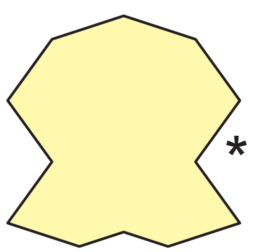
The basic set.



Puzzle A



Puzzle F



Knuth's Pentagons

This puzzle is one of the most fascinating geometrical objects that have ever been created. Donald Knuth was inspired by an idea of Nob Yoshigahara and came up with a unique puzzle set that fully employs its features based entirely on the golden ratio. The set consists of four equally sized pentagons in four different colors each one dissected into four pieces, as shown. There are two sets of similar pieces in each pentagon – two acute isosceles triangles and two obtuse isosceles ones. The pieces of each pentagon form a sequence so that two smaller pieces neighboring in the sequence always can form a bigger piece that is immediately next to that pair.

This puzzle was Stan Isaacs's exchange gift at IPP15 in 1995 in Tokyo, Japan. Several puzzles were proposed with this set: a big pentagon with different constraints, stars made of different selected subsets, and a challenge with a 5-pointed star plus two pentagons. Note that for each of the puzzles (A through T) proposed below all 16 pieces should be used. In the puzzles where two or more shapes are required to be built, they should be done simultaneously, also using a single set of pieces. In some of them pieces of the same color should touch at most at a corner (corner-touch color separation), while in the other puzzles no pieces of the same color should touch even at a corner (total color separation).

Puzzle A. As a warm up assemble a big plain pentagon. Then make it with corner-touch color separation. Finally, form a big pentagon with total separation of colors. Can you find this solution?

This puzzle has been produced since 1995 by Kadon Enterprises, Inc. (www.gamepuzzles.com) under their trademark Puzzling Pentagon™. The set of challenges was greatly elaborated by Kate Jones, and contains dozens of new, wonderful challenges. Try a few of them (Puzzles B thru E).

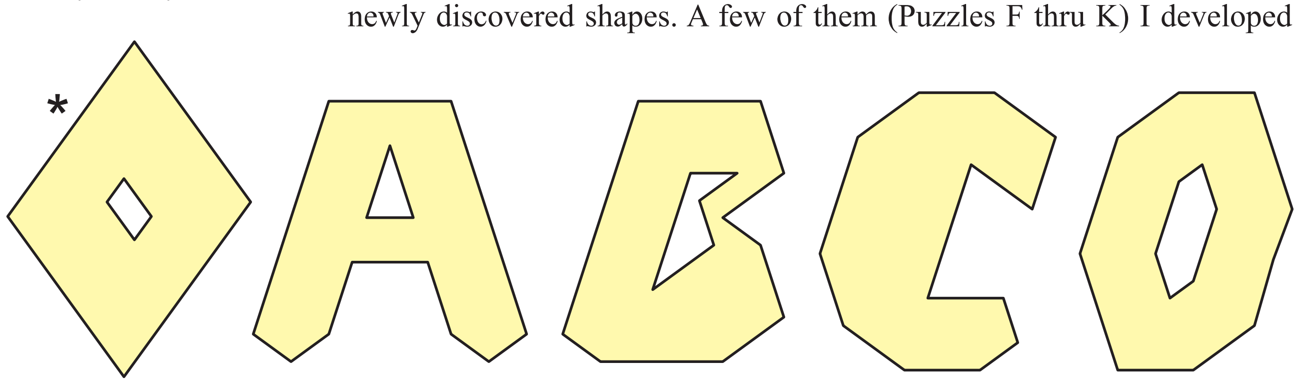
Puzzle B. Assemble four equal pentagons, total color separation.

Puzzle C. Create three different pentagons, total color separation.

Puzzle D. Make a big pentagon so that all four pieces of each color touch and link by corners only.

Puzzle E. Form three concentric pentagons, corner-touch separation.

This puzzle has a magic power – once you start playing with it, it is almost impossible to put it aside. And it still hides many incredible discoveries; besides the initially proposed shapes, you can assemble some newly discovered shapes. A few of them (Puzzles F thru K) I developed



Puzzle R. Create different shapes and letters.

based on some ideas from the Puzzling Pentagon™ set, the remaining are my original challenges. For each of the new puzzles below you have to use the full set of 16 pieces. The total color separation is possible for all puzzles but those marked with asterisks; shapes with * have a corner-touch separation.

Puzzle F. At least seven different parallelograms can be formed; one of them is shown in the opposite page. Can you find them all?

Puzzle G*. Create a big tall trapezoid.

Puzzle H*. Form a big "smashed" trapezoid.

Puzzle I*. Assemble a symmetric convex octagon. What is a convex polygon with the biggest number of corners?

Puzzle J. Make a symmetric star-like shape whose outline consists of alternately repeating peaks and caves (convex and concave angles). Such a shape is called n-pointed star, and it has all of its convex points just acute angles. Find the minimum and maximum* for n.

Puzzle K*. Form a symmetric 5-pointed star.

Puzzle L*. Make two plain, regular, fully symmetric 5-pointed stars.

Puzzle M. Assemble two similar triangles.

Puzzle N. Form two similar rhombuses.

Puzzle O. Create four similar rhombuses.

Puzzle P*. Form two shapes, both similar to that shown above right.

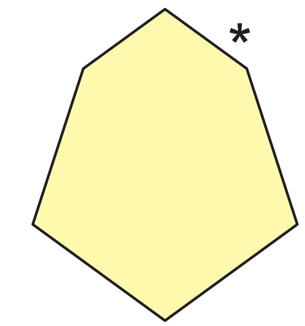
Puzzle Q*. Make a closed fence ("one-piece" wide with a hole) in which adjacent pieces share sides of exactly the same size; see the example.

Puzzle R. Assemble a few shapes* and some letters.

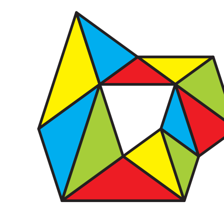
Finally, here are two challenges that can be considered as the hardest of all. These are about triangles similar to unit pieces. There are just two distinctive triangles that can be assembled with the pieces of this puzzle – a big obtuse isosceles triangle and a big acute isosceles one. Each of these shapes has a small opening in it; you will know openings' exact shapes when you accomplish the challenges. The big acute isosceles triangle looks almost impossible, and there are several, "almost" solutions with a smallest piece left and its very tiny overlaps with the neighbors. This gives you a desperate impression that the challenge is just impossible. But it is possible without any tricks!!

Puzzle S. Form a big obtuse isosceles triangle. It will have a small single opening in it.

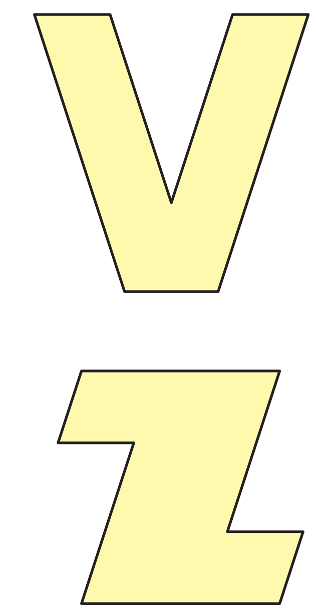
Puzzle T*. Make a big acute isosceles triangle. It will have a small single opening in it.

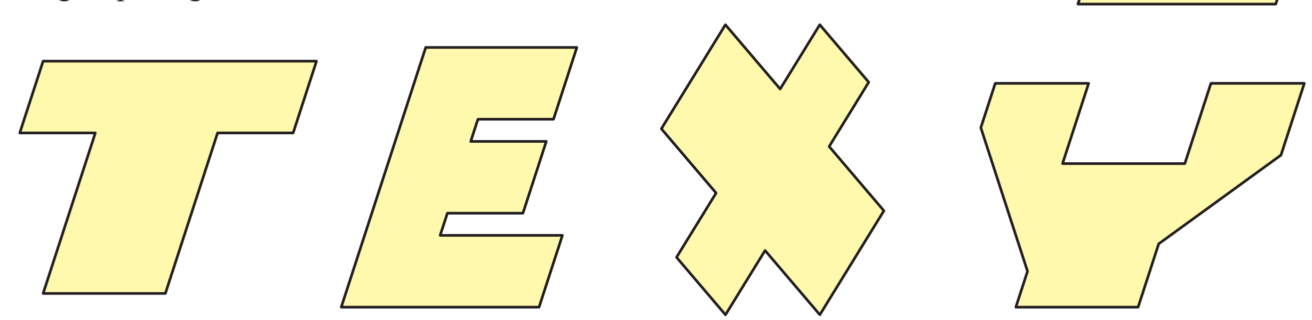


Puzzle P

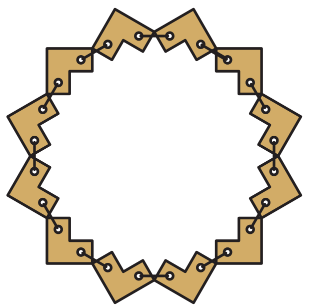


Puzzle Q.
Example with a subset.





Puzzle R (continued). Create different shapes and letters.

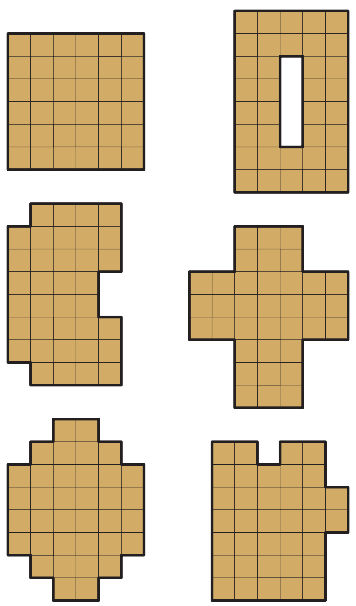


Knuth Necklace
Twelve trominoes make this puzzle.

Knuth's Necklace

Twelve wooden trominoes are connected into a necklace, as shown. Each tromino can independently swing around its neighbors and it can flip over. At the same time, all mutually adjacent unit cells in neighboring trominoes should remain adjacent in every position of these trominoes.

Donald Knuth wore this elegant puzzle at IPP29 in San Francisco, 2009, and his original challenge was about whether it is possible to form a 6 x 6 square with the necklace. Can you solve this challenge? Also, the necklace can produce some other shapes shown in the illustration; just one of them cannot be assembled. Can you solve them all, and show the one which does not belong?



Knuth Necklace
Which of the shapes cannot be formed with the necklace?

The Elephant Tour

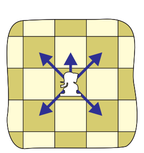
This puzzle on a chessboard deals with a special piece called the Elephant. This piece was used in one of the variants of Shatranj, an ancient predecessor of the modern game of chess. Nowadays the game of chess uses the Bishops in place of the Elephants. The Elephant (or Gaja in Sanscrit) can perform just five short, one-step moves, as shown in the diagram. The four diagonal moves represent the Elephant's legs, and a step forward represents its trunk.

The object of this puzzle is to create an Elephant tour on a given chessboard, not only an 8 x 8 one but also on chessboards of other sizes and shapes. One of the most interesting shapes is an 8 x 8 chessboard minus its four corners. Many closed Elephant tours that visit all 60 cells of this board can be done. And the main challenge is to be very careful with your moves up and down since the Elephant's moving possibility is asymmetric – it has three ways to go up and just two ways to go down. The Elephant's "trunk" moves are very important in the final tours since they allow to change your tour between light and dark cells.

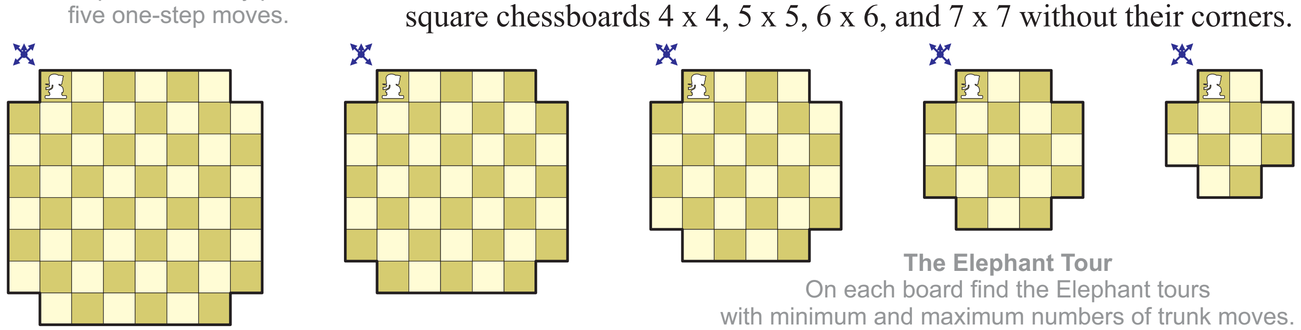
Puzzle 1. Create such a 60-step Elephant tour on the 8 x 8 chessboard without its corners (below on the left) that has the minimum number of trunk moves. Hint. This minimum is less than 5 trunk moves.

Puzzle 2. Create such a 60-step Elephant tour on the 8 x 8 chessboard without its corners that has the maximum number of trunk moves. Hint. This maximum is more than 25 trunk moves.

Puzzles 3-10. The above challenges are not that easy, so you might wish to practice on smaller boards shown below. Again, these are square chessboards 4 x 4, 5 x 5, 6 x 6, and 7 x 7 without their corners.



The Elephant Tour
The Elephant can only perform five one-step moves.



The Elephant Tour
On each board find the Elephant tours with minimum and maximum numbers of trunk moves.

For each shape create a closed Elephant tour that has the minimum number of trunk moves. Then do that with the maximum number of trunk moves. Can you create a closed tour for each of these shapes, or some of them can have just open tours?

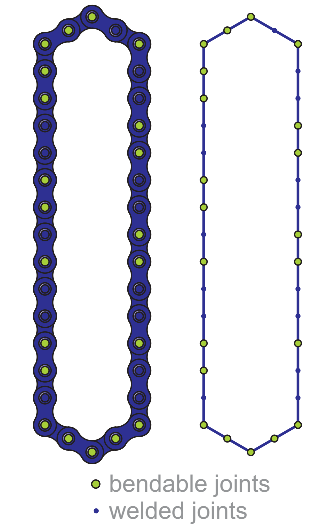
Brain Chain

by Bob Easter and Don Knuth

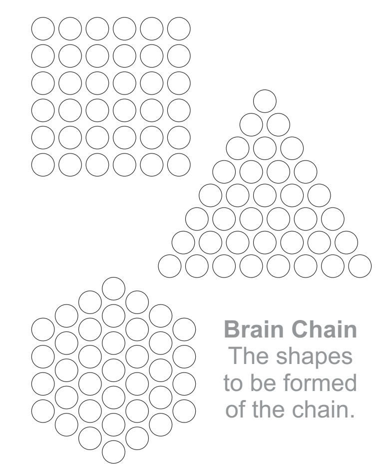
You have a long "bicycle chain" in which some of the joints are welded together and cannot be bent, while other joints are flexible. Knuth developed a pattern of welds so that there are unique solutions to filling both a square and an equilateral triangle, using the fact that 1 + 2 + ... + 8 = 6 x 6. Can you show how to put the whole chain into a square, and then into a triangle? Stan Isaacs used this puzzle as his exchange gift at IPP17 in San Francisco, 1997.

Additional challenge A. If just one of welded joints in this puzzle were flexible, it would be possible to put the whole chain into a right hexagon! Such a hexagon will have a single hole in it. Which welded joint should be bendable to achieve this? Where will the single hole be in this hexagon? And now this chain can also form a rectangle! Find it!

Additional challenge B. What is the maximum number of welded joints for a chain that can form a triangle, a square, and a right hexagon with a hole in its center?



Brain Chain
The chain and its graph.

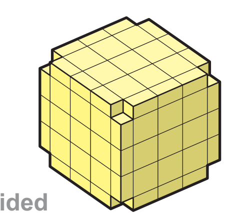


Brain Chain
The shapes to be formed of the chain.

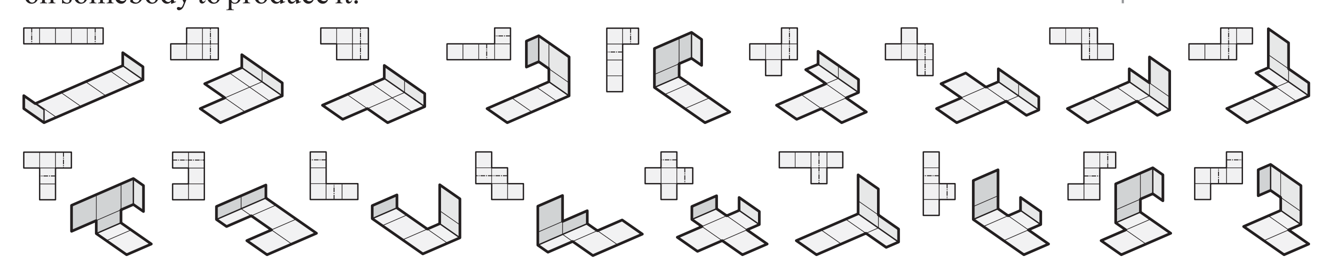
The One-Sided Pentomino Wrap

You have a cube and the eighteen one-sided pentominoes, as shown in the illustration. Each face of the cube is 4 x 4, but the grid is offset by 1/2, so that 3 x 3 cells are in the center of each face. These 3 x 3 cells are surrounded with twelve half-cells. Each one-sided pentomino has two cells that are required to bend around an edge of the cube. Otherwise each pentomino is rigid and flat. We can see the pentominos as made of a thin metal sheet folded permanently where required. The cube should have magnets at each of its fifty-four whole cells (nine magnets per face).

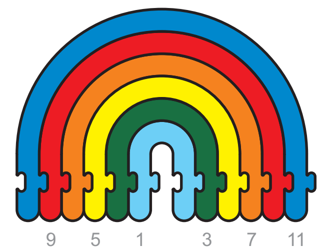
Basically, this puzzle represent a new way to "wrap" the 18 one-sided pentominoes around the faces of a cube. There is a unique solution – can you find it? Note that 1/4-cells at the corners of the cube will left uncovered. Knuth created this puzzle in 2011, and it just awaits on somebody to produce it.

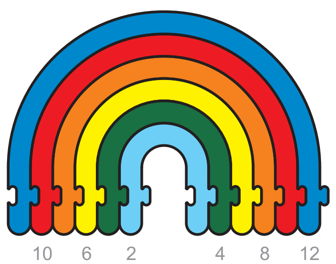


The One-Sided Pentomino Wrap
The cube to be wrapped with the one-sided pentominoes.

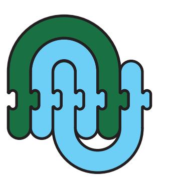


The One-Sided Pentomino Wrap
The eighteen one-sided pentominoes pre-folded for wrapping a 4 x 4 x 4 cube.





Rainbones
The full set of twelve pieces.



Rainbones
The solution for *first* three pieces.

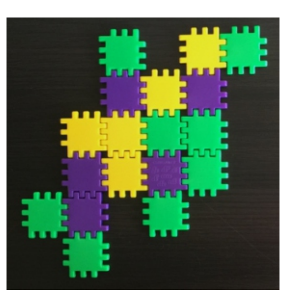
Rainbones

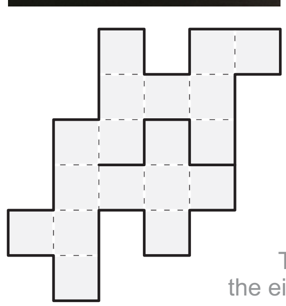
by George Miller and Donald Knuth

In 1958, a Scottish mathematician C. Dudley Langford posed a problem with a set of colored blocks placed in a pile. The problem is described in Martin Gardner's book, *Mathematical Magic Show*. The simplest puzzle is about three pairs of colored blocks, two blocks per color. They are to be placed in a pile (line) so that one block is between two blocks of one color, two blocks are between blocks of a second color, and three blocks are between two blocks of a third color. Later on it was proved that within the first twelve pairs, solutions exists just for three, four, seven, eight, eleven, and twelve pairs. Generally, solutions exist for n pairs if and only if n or n + 1 is divisible by 4. The solutions for three and four pieces are unique. Note that the pile (or line) of blocks should contain no gaps.

The Rainbones puzzle is a clever and colorful presentation of the Langford Problem. It consists of twelve arc-like pieces, as shown. The object is to interlock some set of pieces so that the line of knobs is continuous. A small example is shown for the *first* three pieces. Unfortunately, the Rainbones puzzle's design does not allow solving challenges with *first* four and *first* seven pieces. So, to start, try a simple warm-up puzzle with pieces 1, 2, 4, and 5 only. Then use *first* eight pieces. After that use *first* eleven pieces with the largest piece omitted. Finally, use the entire set of the twelve pieces. Stan Isaacs used this puzzle as his exchange gift at IPP27 in 2007 in Gold Coast, Australia.

Cubigami 7
The straight tetracube shape (above) created of the flexible network (below).

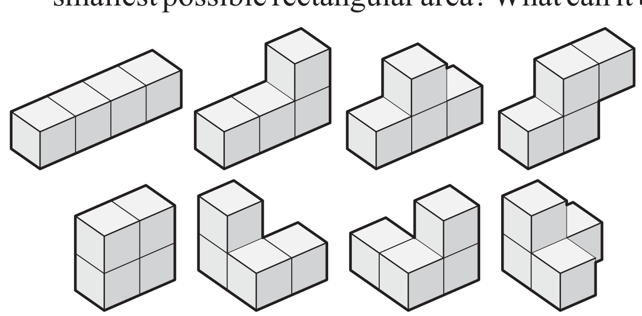




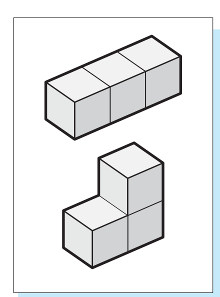
Cubigami 7

by George Miller and Donald Knuth

Folding puzzles are mostly made from paper or thin plastic. Cubigami 7 is an innovative puzzle created by George Miller and Don Knuth. The flexible pattern of eighteen plastic squares stitched together allows us to create seven different tetracube shapes out of the eight shown. Which one cannot be done and why? The one-of-its-kind pattern for this puzzle was developed by Donald Knuth; the pattern can be inscribed into a 6 x 6 area. To understand how difficult such a task is, try to solve just a simpler challenge: Create a flexible pattern which could form both tricube shapes shown below on the right. Can you do it so that it can be inscribed into the smallest possible rectangular area? What can it be – 4 x 4, 4 x 5, or 5 x5?



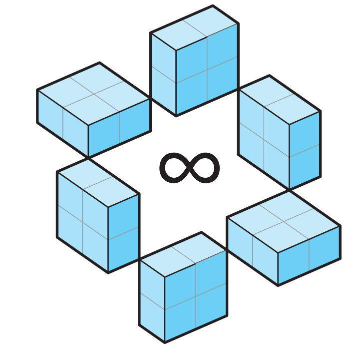
Cubigami 7
The flexible pattern (on the left) can create seven out of the eight tetracube shapes (above). Which one does not belong?



Cubigami 7
Create a flexible pattern for these two tricube shapes.

1 x 2 x 2 Anti-Sliding Blocks

You have infinitely many 1 x 2 x 2 blocks. Put them into three dimensional space so that (i) they are all lined up at integer points, (ii) every face is in contact with some other face, and (iii) 5/16 of the spaces are empty. This puzzle is related to Wil Strijbos's Anti-Slide puzzle which was the 2nd Place Winner at the 6th Hikimi Wooden Puzzle Competition in Japan, 1994. In that puzzle you have to place fewer than sixteen 1 x 2 x 2 blocks into a 4 x 4 x 4 container with a lid so that no block can slide within the container in any direction. There are solutions for 15, 14, 13, and 12 blocks. They have respectively 1/16, 2/16, 3/16, and 4/16 of the spaces empty. As it seems, an 11-block solution is possible just for infinite three dimensional space as proposed by Donald Knuth. Can you find Knuth's solution... or possibly improve it?



1 x 2 x 2 Blocks
Put an infinite number of these blocks into 3D space to reach 5/16 empty.

Perfect Packing

by Donald Knuth and George Miller

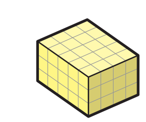
This problem was posed in 1978 by Dean Hoffman. The object is to pack 27 rectangular blocks with dimensions of a x b x c into a cubical box with its edge of a + b + c so that (i) a, b, and c are different, and (ii) the smallest size of the block is longer than (a + b + c) / 4. The smallest dimensions for blocks are 9 x 10 x 11, and a box is 30 x 30 x 30. In 2004, Don Knuth revisited Hoffman's idea, checking the case when the smallest size of the block is equal to (a + b + c) / 4. Now the smallest dimensions for blocks are 3 x 4 x 5, and a box is 12 x 12 x 12. More than that, Knuth discovered that now you can fit into the box not 27 but 28 blocks!! There are three different solutions – easy, medium, and hard. Can you find them all? This puzzle was George Miller's exchange gift at IPP25 in 2005 in Helsinki, Finland.

Knuth's puzzle evokes several new challenges:

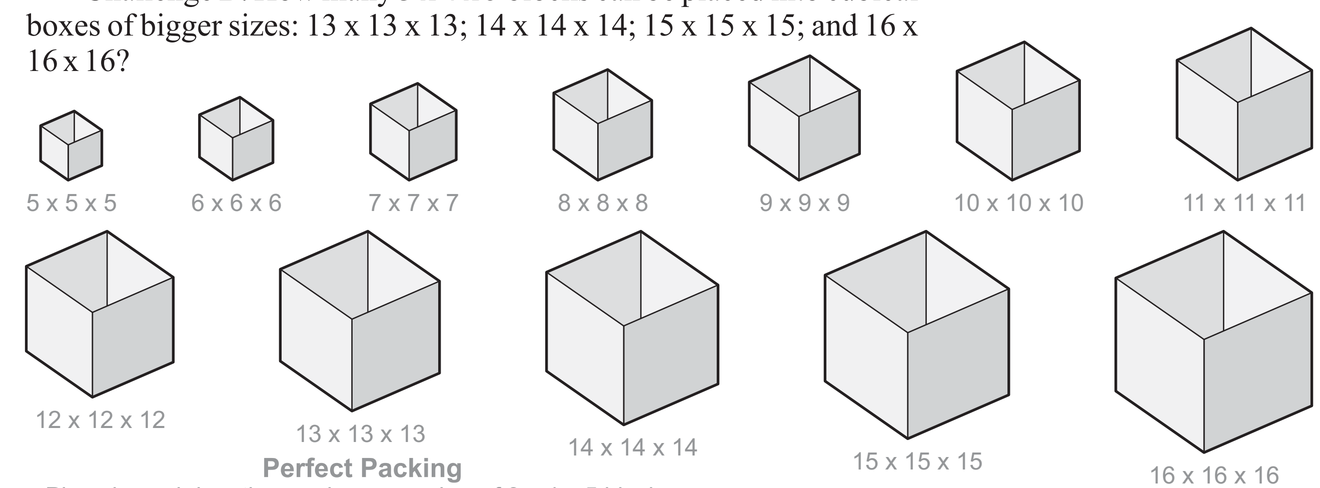
Challenge A. How many 3 x 4 x 5 blocks can be placed into cubical boxes of smaller sizes: 5 x 5 x 5; 6 x 6 x 6; 7 x 7 x 7; 8 x 8 x 8; 9 x 9 x 9; 10 x 10 x 10; and 11 x 11 x 11? The first few cases are trivial.

Challenge B. How many 3 x 4 x 5 blocks can be placed into cubical boxes of bigger sizes: 13 x 13 x 13; 14 x 14 x 14; 15 x 15 x 15; and 16 x 16 x 16?

Perfect Packing
This unique puzzle design was developed and precisely manufactured by George Miller, www.puzzlepalace.com.



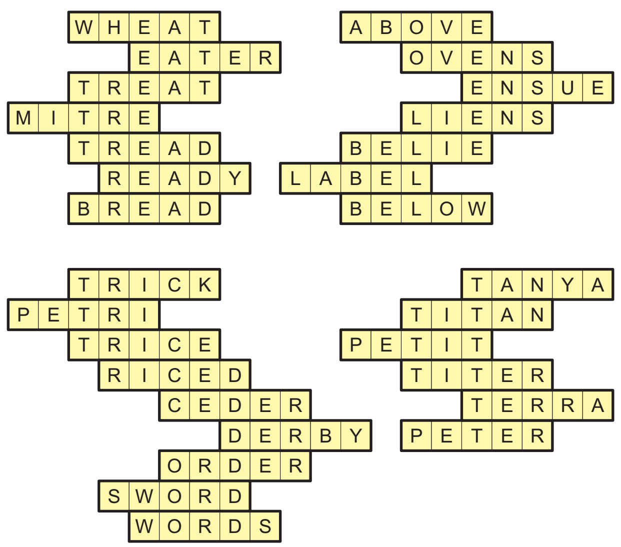
Perfect Packing
A unit, 3 x 4 x 5 block.



Perfect Packing
Place in each box the maximum number of 3 x 4 x 5 blocks.

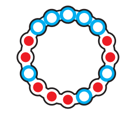
Solutions

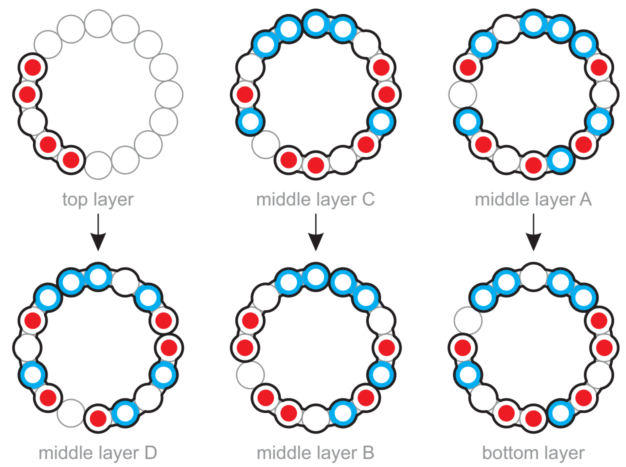
Word Walls



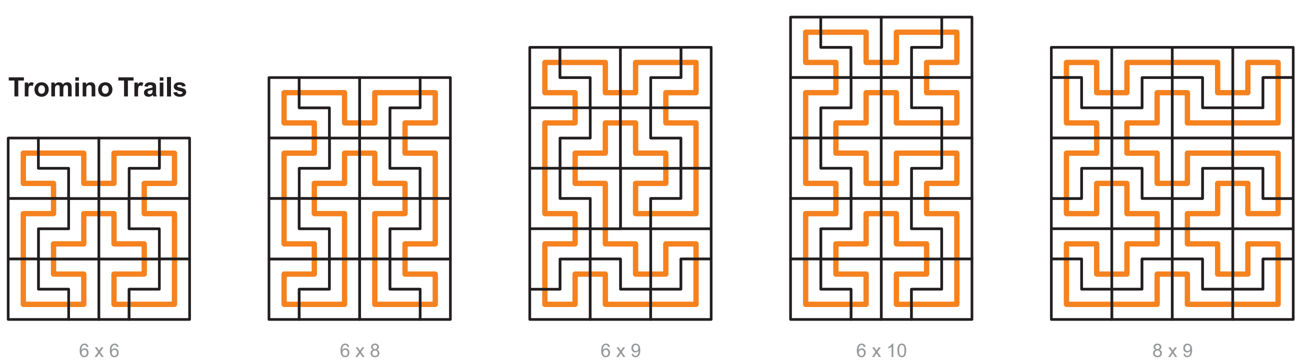
Lucas's Moat

The puzzle has the unique solution, as shown, except for rotation and reflection.



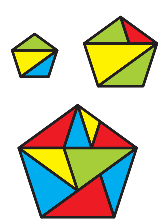


Tromino Trails

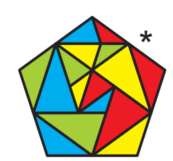


Knuth's Pentagons

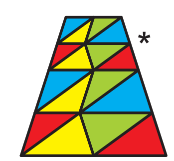
For each shape the currently known best solution is shown below and in the opposite page. The total color separation is possible for all shapes but those marked with asterisks. The marked shapes have a corner-touch separation.



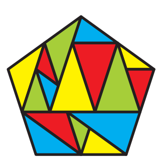
Solution to Puzzle C.



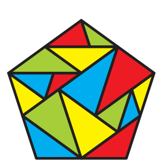
Solution to Puzzles D and E.



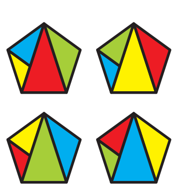
Solution to Puzzle G.



Solution to Puzzle A by Nob Yoshigahara.

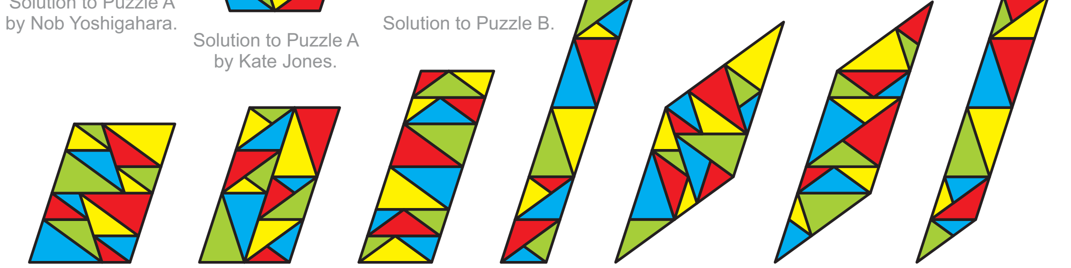


Solution to Puzzle A by Kate Jones.



Solution to Puzzle B.

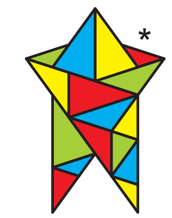
Solution to Puzzle H.



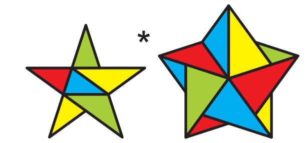
Solutions to Puzzle F.

Solution to Puzzle I.

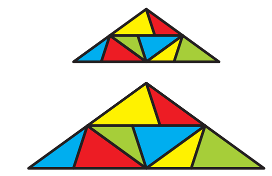
Knuth's Pentagons



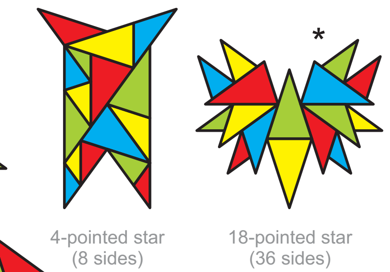
Solution to Puzzle K.

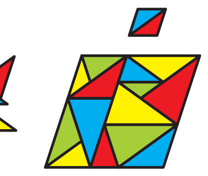


Solution to Puzzle L.

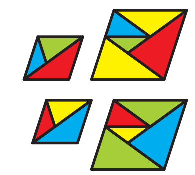


Solution to Puzzle M.

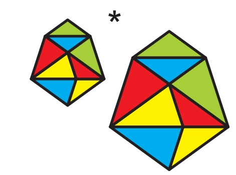




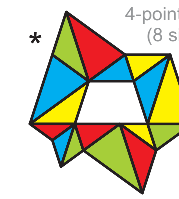
Solution to Puzzle N.



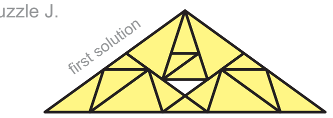
Solution to Puzzle O.



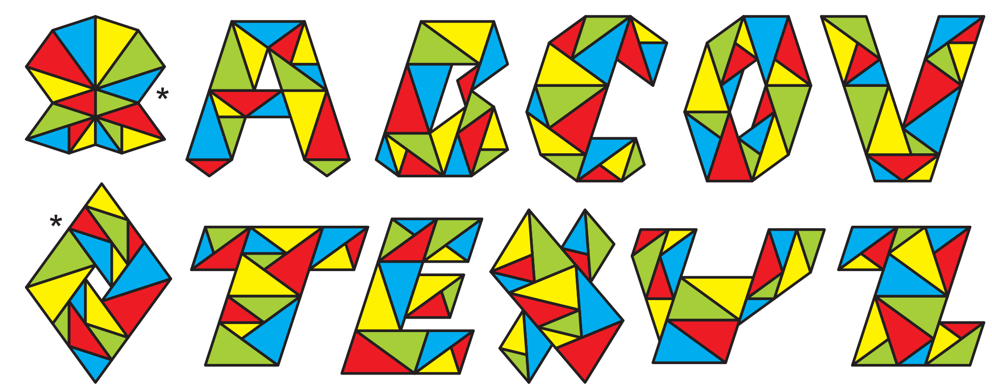
Solution to Puzzle P.



Solution to Puzzle Q.

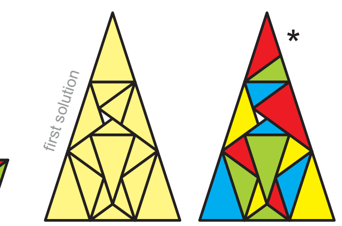


Solutions to Puzzle J.



Solutions to Puzzle R.

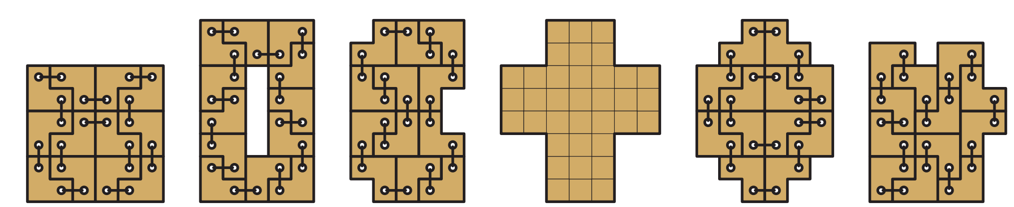
Solutions to Puzzle S.



Solutions to Puzzle T.

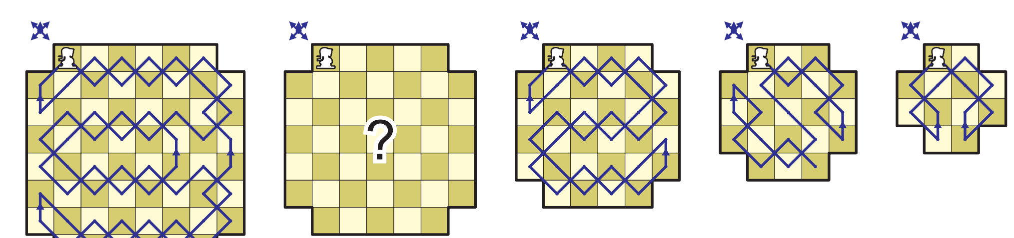
Knuth's Necklace

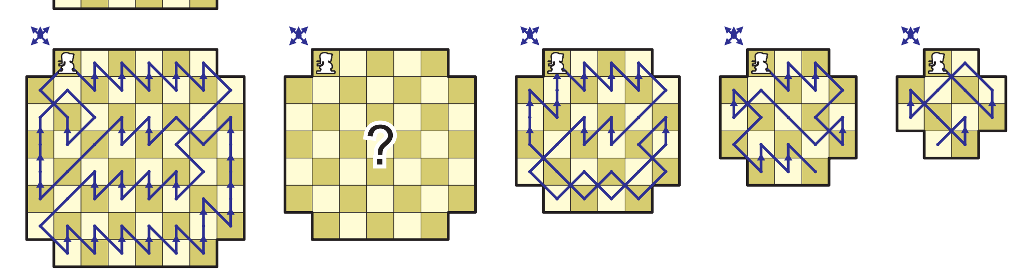
All the shapes but the cross-like one can be formed from the necklace, as shown.



The Elephant Tour

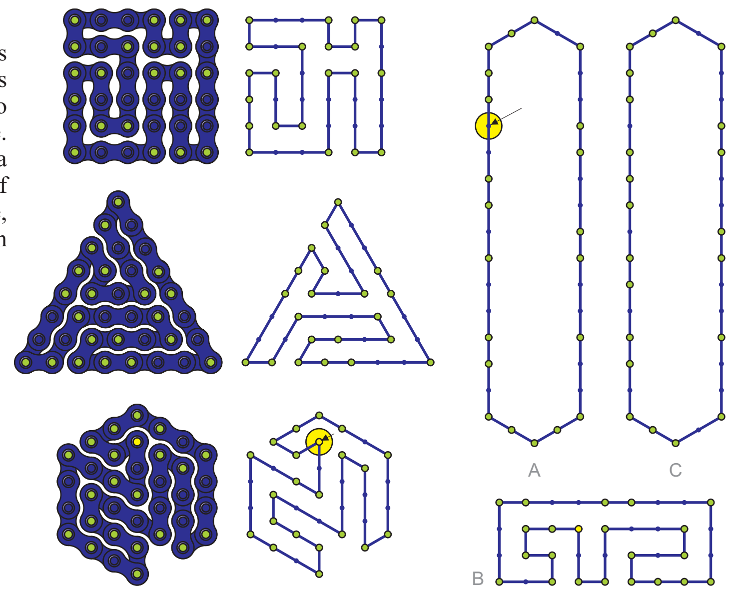
The diagrams on the right show currently known best solutions with the minimum numbers of trunk moves (the top row) and the maximum numbers of trunk moves (the bottom row) for square chessboards 8 x 8, 6 x 6, 5 x 5, and 4 x 4 without their corners. There are no elephant tours on 7 x 7 chessboards! Note that generally in the 8 x 8, 6 x 6, 4 x 4 min-cases one can eliminate any of the trunk moves, because these are closed tours.





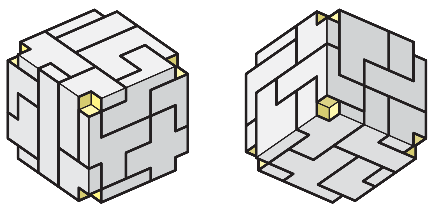
Brain Chain

The solutions for a square and an equilateral triangle are, as shown. If one of welded joints in this puzzle is flexible, as shown in diagram A, it is possible to put the whole chain into a right hexagon, as that shown just below the triangle shape. A single hole is on its periphery. This chain can also form a rectangle 9 x 4, see diagram B. The maximum number of welded joints for a chain that can form a triangle, a square, and a right hexagon with a hole in its center is 14 as shown in diagram C. Can you form those shapes?

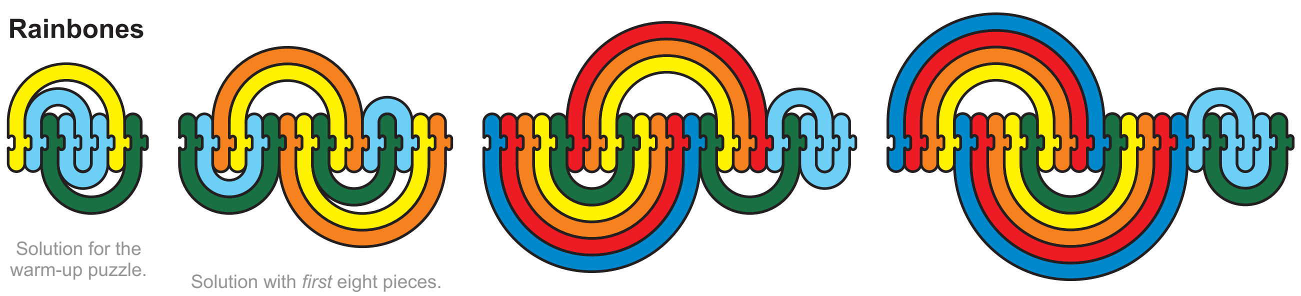


The One-Sided Pentomino Wrap

The unique solution is as shown.



Rainbones



Solution for the warm-up puzzle.

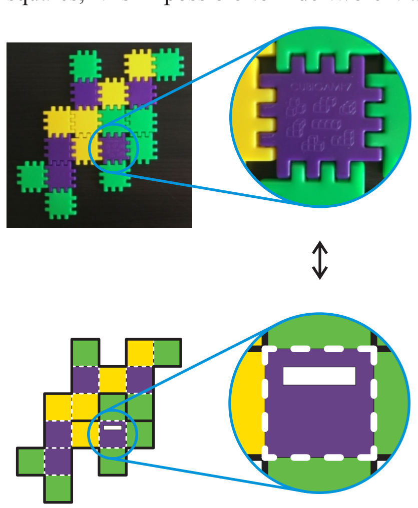
Solution with *first* eight pieces.

Solution with *first* eleven pieces.

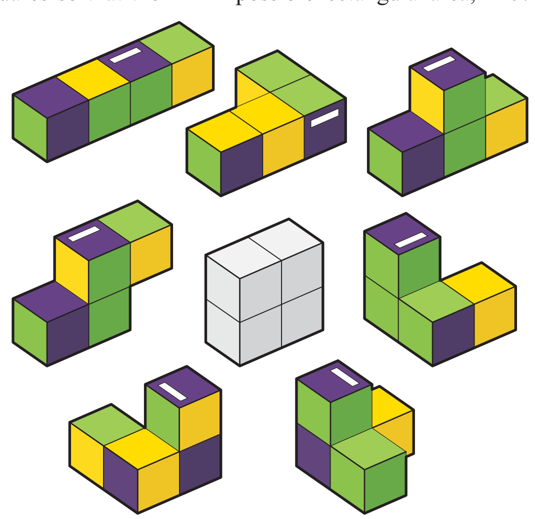
Solution with the entire set of twelve pieces.

Cubigami 7

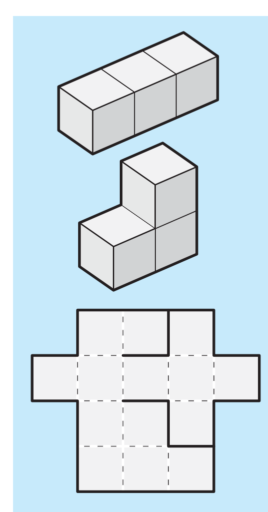
The tetracube shape 1 x 2 x 2, shown in the middle, cannot be done since its surface consists of sixteen unit squares, while the entire flexible pattern consists of eighteen squares. With plastic squares, it is impossible to hide two extra squares so that the whole block looks correct. But it can be done with a paper version. A flexible pattern which could form both tricube shapes is shown below on the right. It can be inscribed into the smallest possible rectangular area, 4 x 5.



Markings on the real puzzle network and on the simplified pattern.



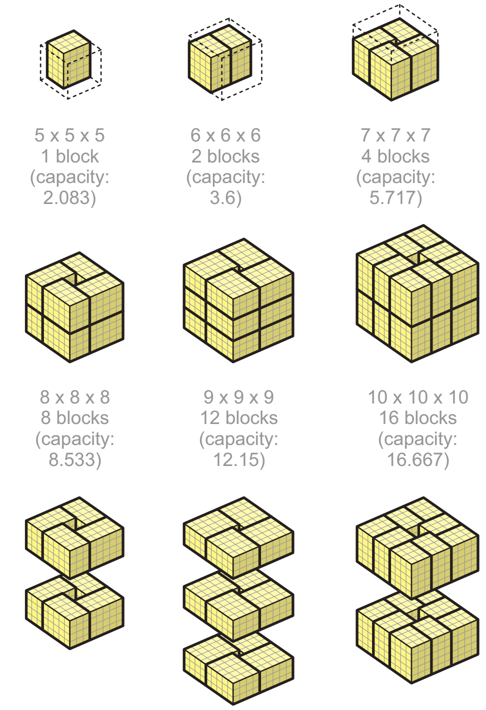
Solutions for seven possible tetracube shapes out of the eight shown.



Two tricube shapes and the network to make them.

Perfect Packing

All, currently known best solutions for cubical boxes in range 5 x 5 x 5 thru 16 x 16 x 16 are as shown.



5 x 5 x 5
1 block
(capacity: 2.083)

6 x 6 x 6
2 blocks
(capacity: 3.6)

7 x 7 x 7
4 blocks
(capacity: 5.717)

8 x 8 x 8
8 blocks
(capacity: 8.533)

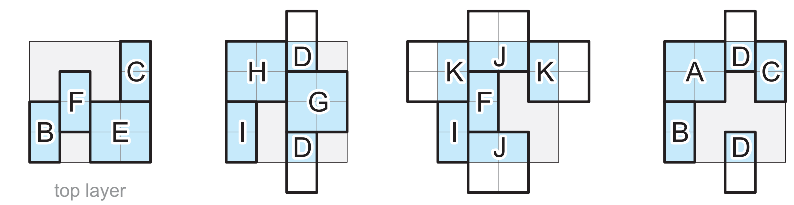
9 x 9 x 9
12 blocks
(capacity: 12.15)

10 x 10 x 10
16 blocks
(capacity: 16.667)

Solutions to Challenge A.

1 x 2 x 2 Anti-Sliding Blocks

One of many possible solutions is shown. A 4 x 4 x 4 pattern is to be repeated endlessly, with 11 blocks and 20 blank spaces, so 5/16 are empty. The blanks form infinitely long caves, not all connected. It is not known if there is a solution with more than 5/16 empty. The original solution by Donald Knuth.

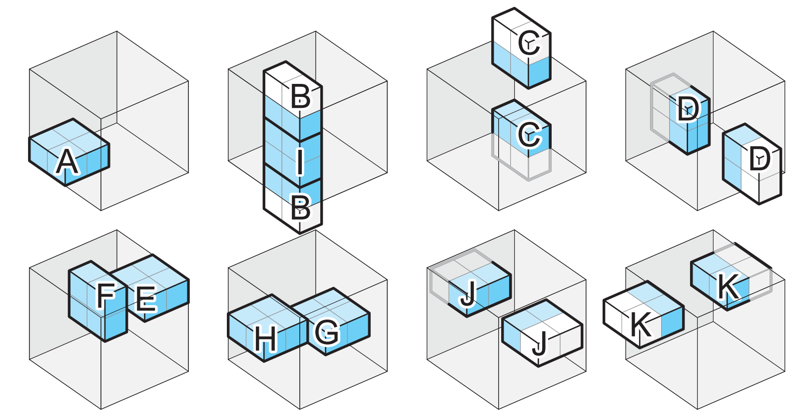


top layer

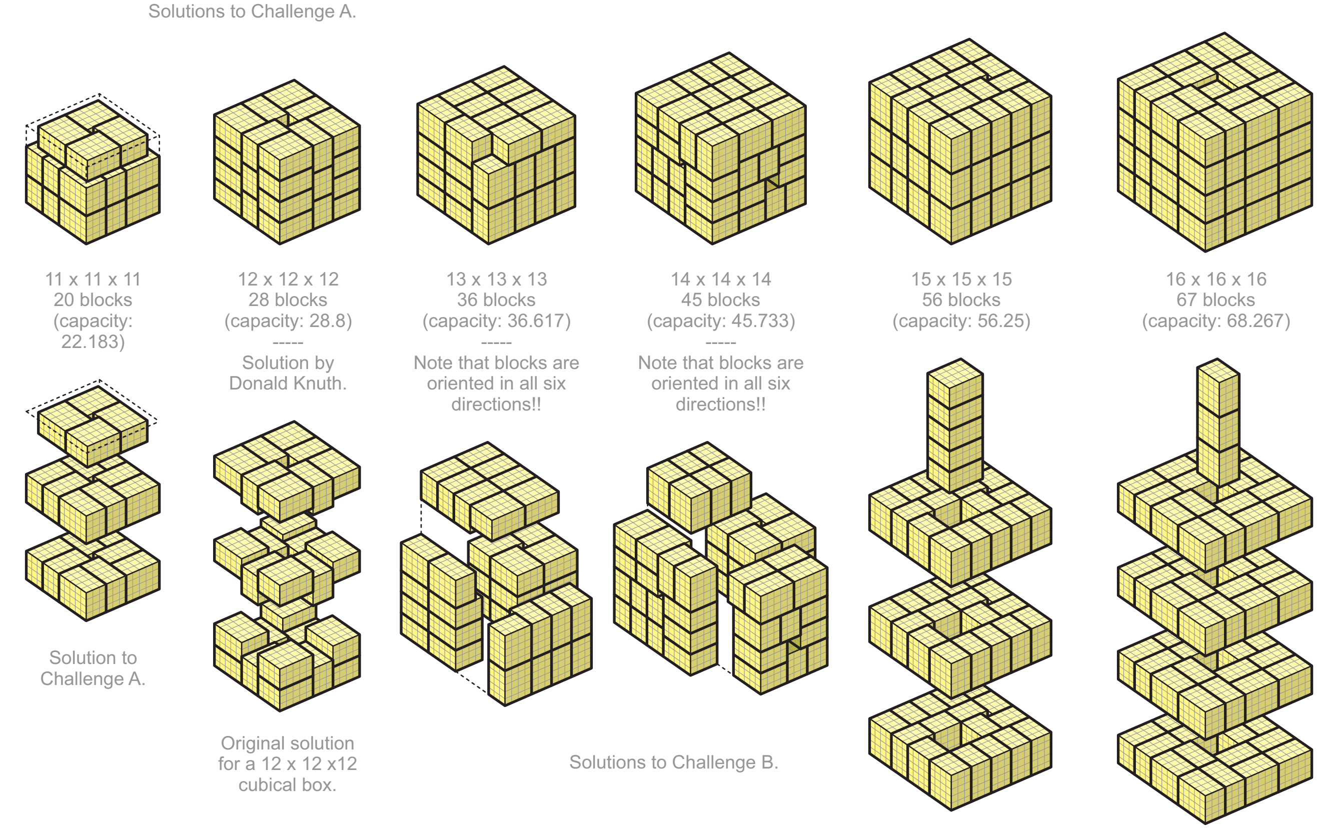
middle layer A

middle layer B

bottom layer



A 4 x 4 x 4 pattern shown layer-by-layer (far above) and with separated blocks (above).



11 x 11 x 11
20 blocks
(capacity: 22.183)

12 x 12 x 12
28 blocks
(capacity: 28.8)

Solution by Donald Knuth.

13 x 13 x 13
36 blocks
(capacity: 36.617)

Note that blocks are oriented in all six directions!!

14 x 14 x 14
45 blocks
(capacity: 45.733)

Note that blocks are oriented in all six directions!!

15 x 15 x 15
56 blocks
(capacity: 56.25)

16 x 16 x 16
67 blocks
(capacity: 68.267)

Solution to Challenge A.

Original solution for a 12 x 12 x12 cubical box.

Solutions to Challenge B.